16752 War

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# PERFORMANCE OF SOME SPREAD SPECTRUM MODES

## Introduction

Spread spectrum modulation has been the subject of much speculation at several of the PRNWG meetings. At various times it has been suggested that the use of a spread-spectrum signal will provide:

greater throughput through time-capture
higher bit-rates in multipath
multiple-channels

and incidentally provide some security advantages.

In TPRN #43 we began an analysis of spread spectrum systems considering only very simple effects. The purpose of this note is to explore the effects of multipath, preamble synchronization, multiple-code (single-channel) operation, and multiple channel operation.

For those readers who are tired of reading so many notes, we begin by summarizing our results and conclusions. Then we present the analysis so that the more careful reader can find the errors.

## Results and Conclusions

We consider first the effects of multipath on a spread-spectrum signal. Multipath reduces the possibility of time-capture or time-interlacing of two packets by stretching the matched-filter output pulse duration. In the presence of moderate to severe multipath spread-spectrum operation will provide very little improvement in throughput over non-spread operation. Table I compares several access modes and shows this clearly. Curves developed later in this note provide more detail.

Next consider the effect of a sync preamble. In the absence of multipath, throughput of an Aloha spread-spectrum system with a sync preamble will approach the throughput of a slotted system. In the presence of multipath a synced Aloha spread-spectrum system will have the

throughput of a slotted Aloha nonspread spectrum system. Table I compares maximum throughput. Details are presented later in a set of curves.

If the transmitted waveform is changed with each bit it is possible to obtain very good performance. Intersymbol interference can be greatly reduced, and multipath can be overcome to a large extent. However, such a system is roughly an order of magnitude more complex than a fixed-waveform system, since many matched filters must be provided. To obtain higher bit rates, transmitter power must be increased to overcome the additional noise accepted by the wider-bandwidth data-rate filter, and to overcome the additional noise of intersymbol interference. For this system, throughput curves in multipath are the same shape as for an Aloha spread-spectrum system with a sync preamble in no multipath. These curves are presented later.

Probably a most interesting result is derived by investigation of multichannel (fixed code per channel) operation. We show that this is an inefficient way to use both transmitter power and rf bandwidth. For example see the last two rows of Table I. Here we contrast several different multichannel schemes. Assume that we have 20 MHz available. Then if we use Aloha terminals with simple DCPSK modulation, we can obtain 30 channels, each with a throughput of $\frac{10^5}{e} \approx 18400$ bits/sec, for a total efficiency of .046 bits/Hz. (This could be increased roughly by a factor of 3 by using QDCPSK and reducing the guard bands.) Using the best spread-spectrum mode we can obtain only 5 channels each with a throughput of $10^5$ bits/sec for a total efficiency of .025. This is only half that of the Aloha system. Furthermore, the spread-spectrum coverage by a single repeater will be much less than the coverage for an Aloha repeater with the same transmitter power. A comparison of multiple Carrier Sense channels and the best spread-spectrum mode, considering multipath effects, is even more enlightening since it is more realistic. Carrier Sense is roughly twenty times as efficient and provides 8/3 times the coverage area for fixed transmitter power.

Table I

COMPARISON OF PERFORMANCE

| | DCPSK (Nonspread) | | | Spread Spectrum N = 100 M = 1000 | | | |
|---|---|---|---|---|---|---|---|
| | Multipath $T_s \leq 5\ \mu s$ | | | No Multipath | | Multipath $T_s \approx 5\ \mu s$ | |
| | Aloha | Slotted Aloha | Carrier Sense | Aloha | Slotted Aloha | Sync Aloha | Slotted Aloha |
| Maximum Through-put | 0.184 | 0.347 | 0.83 | 0.347 | I | 0.22 | 0.5 |
| BW (Hz) | 200 K | 200 K | 200 K | 20M | 20M | 20M | 20M |
| Efficiency (bit/Hz) | 0.092 | 0.184 | 0.415 | 0.00184 | 0.005 | 0.0011 | 0.0025 |
| Efficiency Ratio | 1 | 2 | 4.5 | 0.02 | 0.043 | 0.01 | 0.022 |
| Terminal Receiver Complexity | low | medium | medium | low | medium | low | medium |
| Repeater Complexity | low | medium | low | low | medium | medium | medium |
| No. of Channels Available in 20 MHz | $(1)_{50}$ | $(1)_{50}$ | $(1)_{50}$ | $(2)_6$ | $(2)_6$ | $(2)_6$ | $(2)_6$ |
| Efficiency in 20 MHz BW Bit/Hz | $(1)_{0.046}$ | $(1)_{0.092}$ | $(1)_{0.207}$ | $(2)_{0.011}$ | $(2)_{0.03}$ | $(2)_{0.0066}$ | $(2)_{0.0132}$ |
| Related Area Covered by Fixed Power Transmitter | 1 | 1 | 1 | $(2)_{0.375}$ | $(2)_{0.375}$ | $(2)_{0.375}$ | $(2)_{0.375}$ |

(1) Assuming 100% guard band
(2) Assuming $\alpha = 4$, $\epsilon/R_o = 1/2$, $d_{min}^2 = 10$

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## Effect of Multipath

In TPRN No. 43 we showed that the throughput for a spread-spectrum system using an Aloha access mode with time-capture could be determined from the equation

$$S = Ge^{-G\left(1 + \frac{k}{N}\right)}$$

where S and G have the usual definitions, N is the number of chips per bit in the spread-spectrum signal, and k is determined by the auto-correlation function width of the signal. This equation depends on the assumption that two signals will interfere whenever they are "synchronized" at the receiver so that individual bits arrive within a time window of width $T_w$ where

$$\frac{kT}{2N} = T_w$$

and where T = bit duration (see page 4 of TPRN No. 43). Thus the equation could have been written

$$S = Ge^{-G\left(1 + \frac{2T_w}{T}\right)} .$$

This is valid only so long as $T_w \leq \frac{T}{2}$. When $T_w = \frac{T}{2}$, there is no possibility of "interlace" of two signals and time-capture will provide no improvement over Aloha.

The effect of multipath on any type of signal is to stretch the duration of the signal by echoing it.

For example, if we transmit a very short (300 ns) rf pulse from the hall of Justice in San Francisco we will receive a much longer pulse in the financial district (Professor G. Turin[1] performed this experiment with the results shown in Figure 1). The received pulse appears to be the summation of many of the transmitted pulses, each delayed by a random interval, and each with a random gain and phase. This channel appears to be well modeled by a delay line with random gain taps set at random points.

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(1) G. L. Turin et al., "A Statistical Model of Urban Multipath Propagation," IEEE Trans. on Veh. Tech., Feb., 1972, pp. 1-9.

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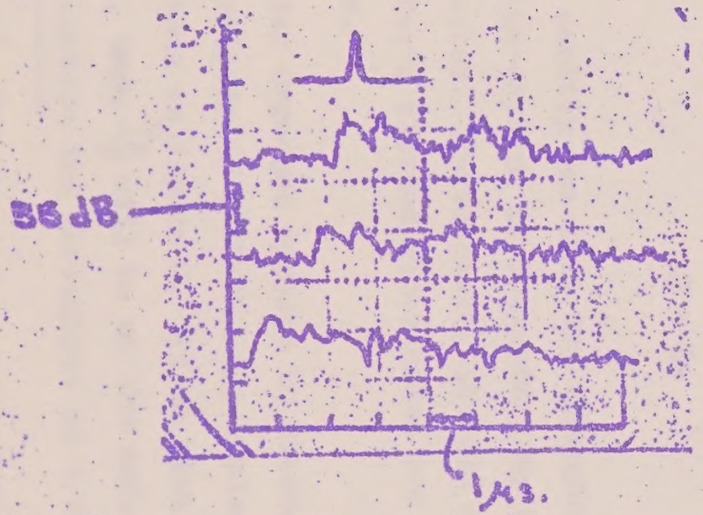


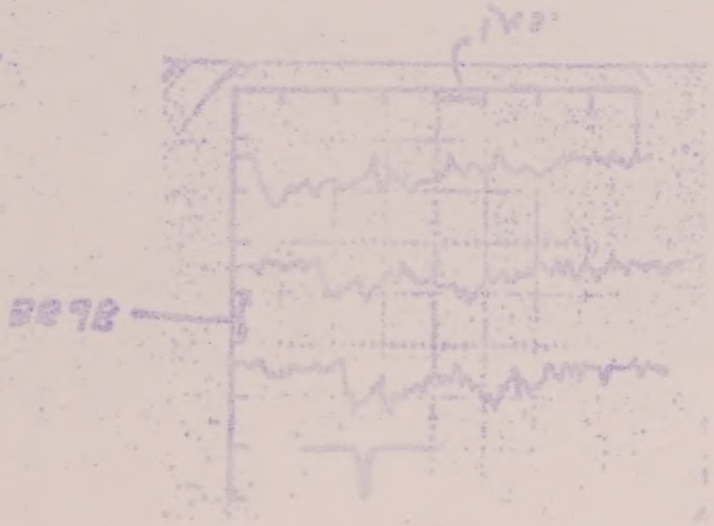

Transmitted Pulse Shape

Received Pulse - 2920 MHz

Received Pulse - 1280 MHz

Received Pulse - 488 MHz

Figure 1 EXAMPLES OF MULTIPATH PROFILES IN SAN FRANCISCO--COPIED AND RETOUCHED WITHOUT PERMISSION FROM TURIN et al., op. cit.



This model is physically appealing, since we can easily visualize a pulse of energy radiated in all directions from the transmitter, and bounding off various surfaces like a beam of light from a mirror. Part of the energy may find a direct path to the financial district, and there are enough reflecting surfaces so that some of them will be oriented to reflect additional copies of the pulse to the district. Notice that Turin's measurements do not show isolated reflections, but indicate a more continuous variation. This is in part due to the fact that the pulse duration is long relative to typical off-path delays; however, it is doubtful that any pulse of reasonable duration (or any other waveform of bandwidth less than a few GHz) would resolve the individual paths. Other measurements made by Turin[1] and Cox[2] support this model, and SRI will be making additional measurements to determine specific parameters of the channel of importance to the packet radio network.

Thus we can see that the effect of multipath on spread-spectrum system performance is to increase the size of the time-window for interference. This time-window will correspond roughly to the 3-dB duration of the impulse-response of the multipath channel. We will call this the delay-spread and designate it $T_s$. Then for reasonably wide-band spread-spectrum signals with bit duration $>2\ T_s$ we have

$$S = Ge^{-G\left(1 + \frac{2T_s}{T}\right)}$$

We can readily see that the throughput of Aloha spread spectrum even with time-capture is determined by the multipath, not by the spreading factor. Furthermore, it will always be less than the throughput for a slotted Aloha mode, and usually be no better than simple Aloha.

A similar argument applies to slotted spread spectrum. If we apply the analysis of TPRM No. 43, replacing $\frac{k}{M}$ by $\frac{2T_s}{T}$ we will obtain curves as shown in Figure 2. If we wish to operate at bit rates of 100 kBps, then $\frac{T_s}{T}$ will be on the order of .2 to 1.0. In this range of operation the throughput of a slotted spread-spectrum system

(2) ibid.

(3) D. C. Cox, "Time-and Frequency-domain Characterizations of Multipath Propagation at 910 MHz in a Suburban Mobile-Radio Environment," Radio Science, Vol. 7, No. 12, Dec. 1972, pp. 1069-1077.

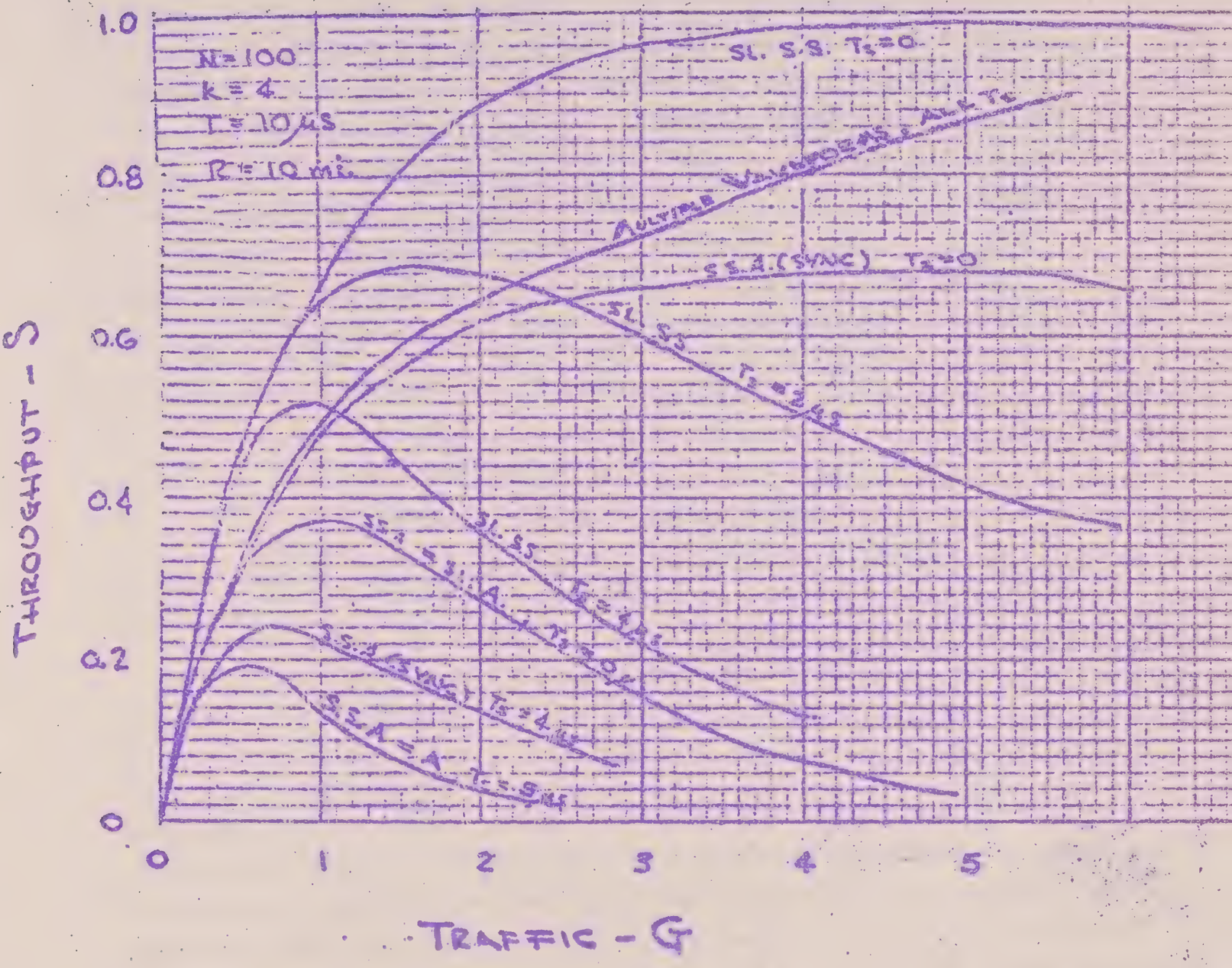
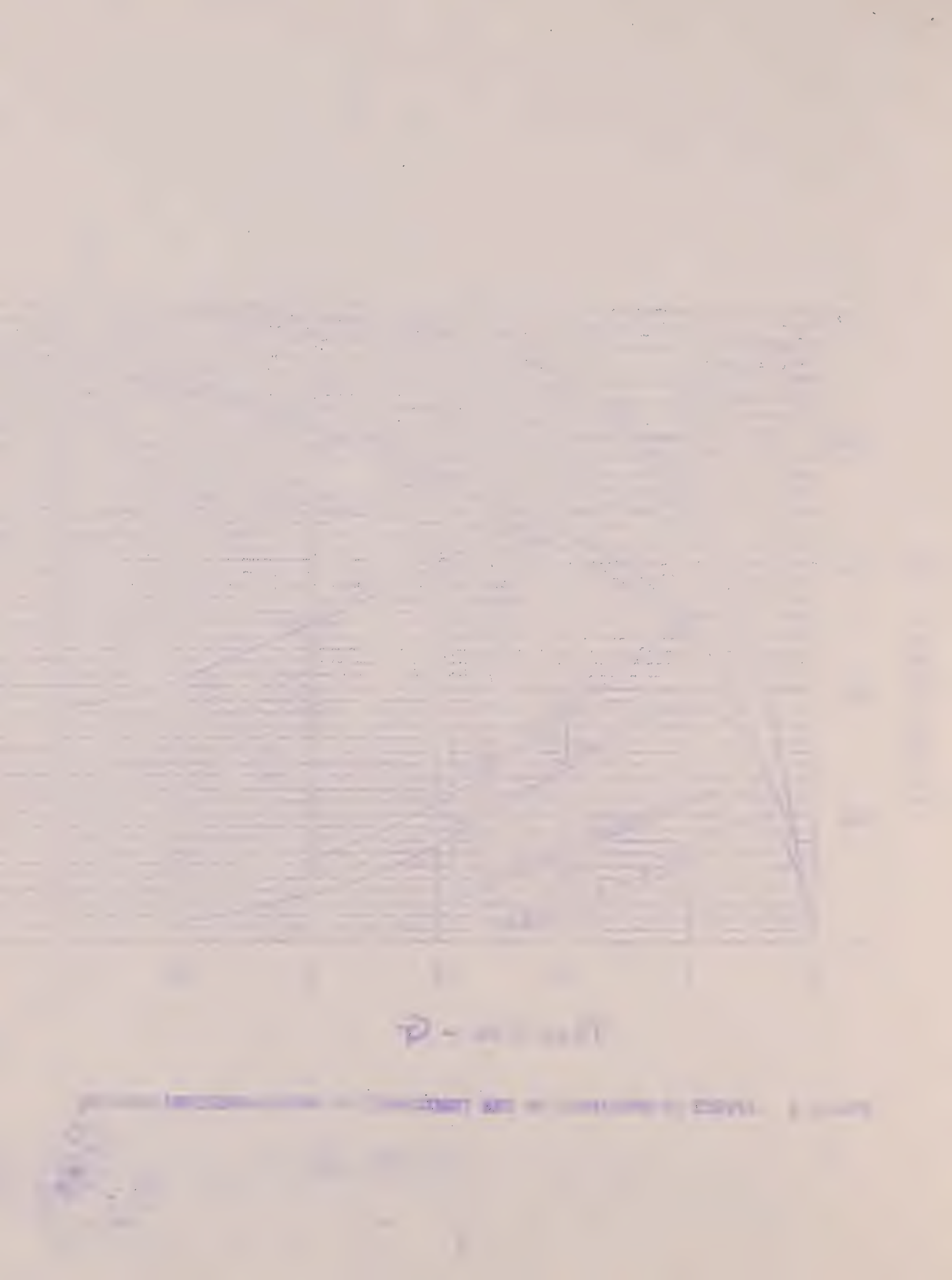



Figure 2 EFFECT OF MULTIPATH ON THE THROUGHPUT OF SPREAD-SPECTRUM SYSTEMS



is considerably better than that of a simple slotted Aloha system, but the price paid for this improvement is a large increase in rf bandwidth.

## Effect of Sync. Preamble

In TPRN No. 43 we assumed that the receiver could be "busy" with useless traffic. This was because we assumed that any signal could capture the bit-sync circuit, so that if two non-interfering packets overlapped the second packet would capture the receiver sync circuit after the first packet released it. In TPRN No. 33, McGuire suggested a sync preamble to precede every packet. Such a preamble has certain associated difficulties since it must be 6 to 10 bits long, and SAWD's of this length may be impossible to produce at bit rates of 100 kBps. Nevertheless, some form of synchronization is possible, and in this case the receiver need never be busy with useless traffic.

We can analyze this case by defining events

C = Sync capture

I = Packet or sync interfered with

A packet will be successful if the event $C\,\bar{I}$ occurs. P(I) will be given by (see TPRN No. 43, p. 6)

$$P(I) = 1 - e^{-\lambda T_m \left(\frac{k}{N}\right)}$$

where $T_m$ = packet duration
(assuming that the sync interference window is no larger than the bit interference window). To determine P(C), consider the sequence of packet start-times as seen by the receiver, after a successful sync capture. No start time prior to $T_m$ sec can capture the sync, and any start time after $T_m$ will capture the sync circuit [although the capture may be interfered with as accounted for in P(I)]. The waiting time after the current packet until the next start time is an exponential random variable with mean $1/\lambda$, hence the average time between successful sync capture will be $T_m + 1/\lambda$. The average time between packet starts is $1/\lambda$, so that the probability of capture is

$$P(C) = \frac{1/\lambda}{T_m + 1/\lambda} = \frac{1}{\lambda T_m + 1} = \frac{1}{G + 1} .$$

The probability of successful packet transmission is

$$P_s = P(C)\,[1 - P(I)]$$

$$= \frac{e^{-G\left(\frac{k}{N}\right)}}{1 + G}$$

and the throughput is

$$S = \frac{G}{1 + G}\, e^{-G\left(\frac{k}{N}\right)}$$

It is interesting to find the maximum throughput possible for large N. This occurs for G such that $\frac{\partial S}{\partial G} = 0$:

$$\frac{\partial S}{\partial G} = \frac{1}{(1 + G)^2} - \frac{k}{N}\,\frac{G}{1 + G}\, e^{-G\left(\frac{k}{N}\right)}$$

has roots at

$$G = \frac{-1 \pm \sqrt{1 + 4\,\frac{N}{k}}}{2} .$$

For large N, $S_{max}$ occurs at

$$G \doteq \sqrt{\frac{N}{k}} ,$$

so that

$$S_{max} \doteq \frac{\sqrt{N/k}}{(1 + \sqrt{N/k})\, \exp\,(\sqrt{N/k})} .$$

for large N

$$S_{max} \doteq \left(\frac{1}{e}\right) \sqrt{\frac{k}{N}} .$$

Thus the addition of a sync preamble to aid time-capture will allow the throughput of spread-spectrum Aloha to approach 1 for large spread-factors if we ignore multipath.

A sync preamble will not change the results for slotted spread-spectrum derived in TPRN No. 43.

Multipath can be accounted for, as before, by replacing k/N by $\frac{2T_s}{T}$ which will degrade performance.

Effect of Multiple Codes (Single Channel)

A possible variation to the spread-spectrum channel follows. Suppose that the transmitter sequences through a large number of codes (one for every bit in the packet). In this case interference between packets occurs only when the packet start-times are synchronized within the time-window of a single bit. The first few bits can act as a sync preamble and the previous results apply, except that the probability of interference is greatly reduced to

$$P(I) = 1-e^{-\frac{G}{M}\frac{k}{N}}$$

where M = number of bits per packet. Thus

$$S = \frac{G}{1+G} e^{-G\frac{k}{MN}} .$$

Notice that even in this case which requires a very complex receiver, throughput cannot exceed $\left(\frac{1}{e}\right)^{k/MN}$.

In this case multipath again limits throughput by broadening the interference window. However, because the code is changed at each bit, interference can only occur in one window out of M. Interfering packets starting in the last M-1 windows will tend to decrease the effective signal-to-noise ratio and may cause occasional errors, however for present purposes we will ignore this problem (but see the next section).

The approach of using a new code for each bit is very much like using a new frequency for each bit (frequency hopping). Transmitters and receivers are roughly of equal complexity for each approach. Bit rates can be achieved which are greater than normally allowed by multipath. Of course higher peak power must be available to achieve higher bit rates.

Multichannel Operation

So far we have restricted the receiver to a single channel so that the results would be comparable to other single-channel schemes. If more capacity is needed, the spread-spectrum format lends itself readily to multiple-channel operation. However, since it requires excess bandwidth and multiple receivers, ultimate performance comparisions can be made only with other multiple-channel schemes.

One of the first questions which arises is how many channels can operate in the same band. The answer is related to the number of codes that can be found with "good" correlation properties, and to the transmitter power and repeater range used. A precise answer has not been found; however an estimate of the number of channels available can be made as follows.

First we note that the channels cannot be truly orthogonal unless all transmitters are in perfect synchronization. In fact, unless the transmitters are in sync to within some very small fraction of a bit, on the order of T/N, all channels will be interfered with by all others.

To obtain some quantitative results, suppose that the codes for each channel are generated as small segments of a very long maximal-length sequence. Let N = number of chips per bit and $N_c$ = number of channels. Consider the matched filter output of one channel. The performance of this channel (in terms of bit error rate) will depend on the probability distribution of the matched filter output. This output will consist of a desired component, due to the signal on the desired channel and undesired components caused by noise and interfering channels. For the moment ignore interfering packets on the desired channel. The matched filter output, sampled at the bit synchronous moment, can be represented as

$$Z = \sqrt{E_d} + n + V_I$$

where $E_d$ = energy received in desired channel per bit

$n$ = noise sample

$V_I$ = interference voltage sample.

The interference is made up of the sum of several signals of random phase and amplitude, on the competing channels. To the extent that the channels are independent, and that there are many of them, we can invoke the central limit theorem to assume that $V_I$ is Gaussian. If the noise is also Gaussian, then Z will be a Gaussian random variable, and the bit error rate of bit decisions based on Z will be determined by the generalized signal-to-noise ratio $d^2$:

$$d^2 = \frac{E(Z)^2}{var(Z)}$$

In the case at hand

$$d^2 = \frac{E_d}{N_o + \overline{V_I^2}}$$

Because of the pseudo-random nature of the coding waveforms, the interfering energy of one waveform will vary inversely with the number of chips per bit, N. There will be $N_c$ of the interfering waveforms, so that

$$\overline{V_I^2} = \frac{N_c}{N} \overline{E_I}$$

where $\overline{E_I}$ = average energy per bit of interfering channel considering duty-cycle and range variations.

To evaluate $\overline{E_I}$, assume that each terminal radiates energy received at the repeater which is given by $\frac{E_t}{r^4}$ where $r$ = range to terminal. Suppose

$E_d$ = energy received in desired channel per bit

$n$ = noise sample

$V_I$ = [illegible] voltage sample

that the terminals are uniformly distributed in a ring about the repeater with minimum range $\epsilon$ and maximum range $R_o$. Notice that $\epsilon$ cannot be zero. Then the average power, when the channel is on, is

$$E\left\{E_I \mid ON\right\} = \int_{\epsilon}^{R_o} \frac{E_t}{r^4}\, p(r)\, dr$$

where $$p(r) = \frac{2r}{R_o^2 - \epsilon^2} \qquad \epsilon \le r \le R_o$$

$$= 0 \qquad \text{else}$$

so that

$$E\left\{E_I \mid ON\right\} = \frac{E_t}{R_o^2 \epsilon^2}$$

If S = individual channel throughput then the duty-cycle is bounded by that S so that

$$\bar{E}_I \ge \frac{E_t S}{R_o^2 \epsilon^2}$$

Thus for a terminal at $R_o$

$$d^2 \le \frac{E_t / R_o^4}{N_o + N_c \frac{E_t S}{R_o^2 \epsilon^2}}$$

For acceptable bit error rate, the system must be designed so that $d^2 > d_{min}^2$ where $d_{min}^2$ depends on the modulation type, but $d_{min}^2 \cong 10$. Suppose that the system is designed so that the $d^2$ at $R_o$ is equal to some multiple ($\alpha$) of $d_{min}^2$ in the absence of interference (this is an inefficient design, but necessary for multichannel operation).

Then

$$\frac{E_t}{N_o R_o^4} = \alpha\, d_{min}^2$$

and

$$N_c \le \left(\frac{\alpha-1}{\alpha}\right)\left(\frac{\epsilon}{R_o}\right)^2 \frac{N}{8d_{min}^2}$$

Thus the number of channels which can operate simultaneously in the same band depends on two parameters which measure the inefficiency of power utilization. The first is $\alpha$, which is the ratio of terminal power needed when operating multichannel to power needed when operating single-channel. This parameter must be greater than one for multichannel operation, thus multichannel operation requires inefficient use of power (this result applies only to code-division multichannel operation).

The second parameter is $\frac{\epsilon}{R_o}$, the ratio of minimum repeater-terminal range to maximum repeater-terminal range. The minimum range must be a significant fraction of the maximum range for multichannel operation. In some geometries this is no limitation, as for example when the repeater is on a mountain at some distance from the area being served. However, when the repeater is surrounded by terminals, $\epsilon$ will be necessarily small, so that either $R_o$ will also be small, or $\frac{\epsilon}{R_o}$ will be small, and multi-channel operation will be impossible.

Thus we may conclude that multichannel operation of a packet radio net using code-division to obtain multiple channels is an inefficient way to allocate power.

Now consider bandwidth efficiency, $\eta$.

$$\eta = \frac{N_c}{N} S$$

where $S$ = throughput of an individual channel. Clearly $\eta$ is bounded by

$$\eta \leq \frac{\alpha-1}{\alpha}\left(\frac{\varepsilon}{R_o}\right)^2 \frac{1}{d_{min}^2}$$

In the limit of power inefficiency ($\alpha \to \infty$) and when all the repeaters are at the same range $\eta$ will be bounded by $\frac{1}{d^2}$ or roughly 1/10. For any practical system, $\eta$ will be considerably less than 1/10.

We may conclude that code-division operation is an inefficient way to use either power or bandwidth.

Coverage Areas

It is interesting to compare the coverage area which can be obtained using multichannel spread spectrum to that obtained using multichannel frequency division for a fixed transmitter power.

Suppose that the transmitter power is such that the received signal-to-noise ratio (without interference) is $d_{min}^2$ at R. Then the area of coverage for a frequency-division scheme is

$$A = \pi R^2$$

For multiple codes, we have shown that the range at which $d_{dim}^2$ will be achieved is

$$R_o = \frac{R}{\alpha^{1/4}} .$$

Also, no terminals may reside inside a circle of radius $\varepsilon$. Therefore, the multichannel coverage area is

$$A_m = \pi R_o^2 - \pi \varepsilon^2 ,$$

or

$$A_m = \frac{A}{\sqrt{n}} \left(1 - \frac{e^2}{R_o^2}\right)$$

## Conclusions

Spread spectrum does not seem to be the best choice for an initial packet radio net unless other considerations outweigh the efficiency arguments made in this note.